FIFO policy for Perishable Inventory systems under fuzzy environment

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Abstract

This paper studies the performance characteristics of perishable inventory systems with First In First Out (FIFO) selection policies in Fuzzy environment. It is assumed that the system has Fuzzy Poisson arrivals and Fuzzy Poisson demand epochs with known rates and non- deterministic shelf life. The performance characteristics derived for FIFO policy are spoilage rate, loss sales rate, mean time between stockouts, inventory level on hand and the distribution of age of items delivered. In particular, the dependence of these characteristics on the replenishment rate, demand rate and shelf life are evaluated theoretically.

Index Terms— Perishable inventory, First in first out, fuzzy poison arrival, fuzzy demand rate, fuzzy shelf life time, customer separation time, the forward recurrence time.

1 INTRODUCTION

Many Consumable products such as food items, Pharmaceuticals , flowers etc, have a short lifetime. These products not only generate a substantial amount of revenue but also derive store traffic. Studies shows consistently that customers choice of Departmental stores is influenced heavily by what perishable goods are on their shelves(Tsiros and Heilman 2005).

Inventory management of perishable goods has a long history in the operations literature. Nahmias (1982) provided a review of early work. Recent reviews by Karaesmen et al. (2008) and Nahmias (2011) indicate a considerable renewed interest in the area.

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The first evaluative model of perishable systems is presented by Pegeles and Jelmert (1970). Their objective is to figure out the effects of the issuing policy on the average inventory level and on the average age of the issued items. Brodheim et al (1975) developed a model of a system with scheduled deliveries of a fixed amount. A constant inventory level is considered by Chazan and Gal (1977). Graves (1982) developed the steady state stock distribution of a constant replenishment inventory system with compound Poisson or unit demand requests. And as an extension of the above Nahmias (1982b) uses the results of his study to determine the replenishment rate which minimizes the total operating costs per time unit. Kaspi and Perry (1983,1984) computed the generating function of the limiting distribution of the number of items in certain FIFO systems having unit input process and poisson demand.

The study on management of Perishable goods is found challenging due to their high

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obsolescence or perishability costs. The problems arising in the management of special consumable food materials in the departmental stores have been a motivation for this study. These highly specialized items include for example fresh vegetables, milk products etc. The amount of perishable goods thrown away by retailers is alarmingly high and has come under continuous public scrutiny in recently years (Bloom 2010, Stuart 2009). Compliance with stand age control requirements can limit the shelf life of these items to anywhere between a week or two. Long term supply contracts from multiple sources are established to assure the desire availability and quality levels. Hence the arrival rate of such products are often random. Moreover the demand pattern for these items are also found to be random. In such cases where the arrival and demand rate are random determination of the distribution of the stock level is difficult because such evaluation must include the stock level at every time epochs.

This study first characterize the stock dynamics needed to evaluate various performance measures for the FIFO policy. Using these results one can choose the desired replishment rate and could use to consider the purchase, service level, tax and inventory holding cost.

2. The Model Description:

Notations:

- $\tilde{\lambda}$- fuzzy arrival rate
- $\tilde{\mu}$ -fuzzy service rate
- $\tilde{D}$- fuzzy shelf life time
- $\tilde{T}_{Cj}$ - customer separation time of the jth customer
- $\tilde{T}_F$ - the forward recurrence time of the time $\tilde{T}_C$
- $\tilde{X}$ **(t)** - the anticipated time to selection *of that item* if no other items were to come, given that at time **t** that item has not been selected.

2.1 Assumptions:

The perishable inventory system of interest has the following characteristics:

- The arrival of fresh items follows a Poisson process with mean λ. arrivals per unit time.
- The mean demand rate is μ requests per unit time.
- Each demand request is for one unit at a time.
- Demand requests arriving when the inventory system is empty are lost.
- The stored items have a shelf life of D time units.
- The demand process is a Poisson process independent of the arrival process.
- An item which is not used to meet a demand request within D time units perishes (exits the system).

2.2 The FIFO Inventory System

Consider the perishable inventory system with fuzzy poisson arrivals to inventory of rate $\tilde{\lambda}$ and deterministic shelf life $\tilde{D}$. Again one has a sequence of demand epochs with i.i.d. separation

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times $\tilde{T}_{Cj}$ and associated demand rate $\tilde{\mu}$ = E [$\tilde{T}$j] . The intervals between arrivals will be designated by $\tilde{T}_{Aj}$ and these too are identically independently distributed (i.i.d)with expectation $\tilde{\lambda}^{-1}$. At each demand epoch, if the inventory is not empty, the oldest item is chosen. The system is said to be FIFO in that items which have not perished are delivered on a first-in first-out basis.

2.3 The age $\tilde{X}$(t) of the oldest item

The process describing the number of items in the system is not fuzzy Markovian and is not tractable. Instead one works with the age $\tilde{\boldsymbol{X}}$**(t)** of the oldest item using the convention that $\tilde{\boldsymbol{X}}$**(t) = 0** when the system is empty.

Theorem 1: The process $\tilde{\boldsymbol{X}}$ **(t)** is Fuzzy Markov for FIFO discipline.

Proof:

The process $\tilde{\boldsymbol{X}}$ **(t)** is Fuzzy Markov, for the five following reasons:

- when an item of age $\tilde{D}$ perishes, the age of the oldest item becomes **max[0,** $\tilde{D}$ **-** $\tilde{T}_{Aj}$] where $\tilde{T}_{Aj}$ is the arrival lag between that item and the next arrival
- when an item of age $\tilde{\boldsymbol{X}}$< $\tilde{D}$ is selected, the age of the oldest item becomes max[0, $\tilde{\boldsymbol{X}}$ - $\tilde{T}_{Aj}$]
- when $\tilde{\boldsymbol{X}}$ (t) = 0, an arrival gives rise to age 0+;
- between demand epochs, $\tilde{\boldsymbol{X}}$ (t) increases at rate 1
- there is a constant hazard rate $\tilde{\lambda}$ for an arrival epoch and hazard rate $\tilde{\mu}$ for a demand epoch. The process $\tilde{\boldsymbol{X}}$ **(t)** is therefore Markovian. □

It has been noted that when $\tilde{D}$ is finite, the process $\tilde{\boldsymbol{X}}$ **(t)** is ergodic even when $\tilde{\lambda} > \tilde{\mu}$. Its state space $\tilde{N}$ is the union $\tilde{N}$ = {$\tilde{S}$ ⊕ $\tilde{\boldsymbol{E}}$ } of the set $\tilde{S}$ = {($\tilde{s}$, $\tilde{x}$) : 0 < $\tilde{x}$ < $\tilde{D}$}where stock is available and of the point state $\tilde{\boldsymbol{E}}$ for stockout. The motion on the state space $\tilde{N}$ is as follows. There is a constant hazard rate $\tilde{\lambda}$ for new arrivals. If the stock is not empty there is a drift to the right due to aging at unit velocity. At demand epochs, the oldest item is removed and the age of the oldest item jumps to the left by x with pdf $\tilde{\lambda}$ exp(-$\tilde{\lambda}$ $\tilde{x}$). If the virtual value of $\tilde{\boldsymbol{X}}$ **(t)** after a jump is negative, $\tilde{\boldsymbol{X}}$ **(t)** enters the idle state $\tilde{\boldsymbol{E}}$**.** When $\tilde{\boldsymbol{X}}$ **(t)** reaches $\tilde{D}$, the item is removed and $\tilde{\boldsymbol{X}}$ **(t)** jumps to the left by x with pdf $\tilde{\lambda}$ exp(-$\tilde{\lambda}$ $\tilde{x}$), etc'. In the idle state ***E*** there is hazard rate $\tilde{\boldsymbol{X}}$ for transition to ($\tilde{S}$,O+). The motion of $\tilde{X}$(t), on the state space $\tilde{N}$ is shown in Figure 1.

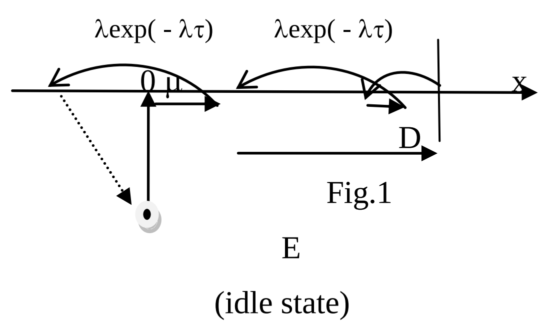

Figure 1

2.4 Evaluation of the distribution on the state space

If one looks at the motion of $\tilde{\boldsymbol{X}}$ **(t)** on the set $\tilde{S}$ **=** {($\tilde{s}$, $\tilde{x}$) : 0 < $\tilde{x}$ < $\tilde{\boldsymbol{D}}$**},** it is seen that this motion, apart from the influence of the two boundaries at $\tilde{x}$ = 0 and $\tilde{x}$ = $\tilde{\boldsymbol{D}}$, consists of exponentially distributed

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jumps to the left with hazard rate $\tilde{\mu}$ and uniform drift to the right at rate unity. Let the spatially homogeneous process in the absence of boundaries starting at t = 0 be designated by $\tilde{\boldsymbol{X}}_H(\mathbf{t})$.

This process has a generalized dynamic Green density

$$g_H(x,t) = \frac{d}{dx} P\big(\tilde{\boldsymbol{X}}_{\mathbf{H}}(t) < x \,\big|\tilde{\boldsymbol{X}}_{\mathbf{H}}(0) = 0\big)]$$

with Laplace transform

$$\gamma_H(W,t) = E\boldsymbol{[exp}\big(\tilde{\boldsymbol{X}}_{\boldsymbol{H}}(t) < \tilde{x}\big|\tilde{\boldsymbol{X}}_{\boldsymbol{H}}(0) = 0\big)]$$

$$= \sum_{k=0}^{\infty} e^{-\tilde{\mu}t} \frac{(\tilde{\mu}t)^k}{k!} \left(\frac{\tilde{\lambda}}{\tilde{\lambda}-w}\right)^k e^{-wt}$$

$$= exp\left(-\tilde{\boldsymbol{\mu}}t\left(1-\frac{\tilde{\lambda}}{\tilde{\boldsymbol{\mu}}}\right) - wt\right)$$

Correspondingly one has the ergodic Green density transform

$$\gamma_H(W,\infty) = \int_0^{\infty} \gamma_H(W,t)dt = \frac{1}{\left(\tilde{\boldsymbol{\mu}}\left(1-\frac{\tilde{\lambda}}{\tilde{\boldsymbol{\mu}}}\right)+w\right)}$$

The compensation measure needed to recreate the influence of the two boundaries is obtained as follows. When an item reaches age $\tilde{\boldsymbol{D}}$, it is discarded and is replaced by $[\tilde{\boldsymbol{D}} - \tilde{\lambda}^{-1}\tilde{\boldsymbol{E}}]$, where $\tilde{\boldsymbol{E}}$ is the exponential variate with mean value unity. This is equivalent to an injection of negative mass concentrated at $\tilde{\boldsymbol{D}}$ to annihilate the item and to injection of positive mass having the distribution of $[\tilde{\boldsymbol{D}} - \tilde{\lambda}^{-1}\tilde{\boldsymbol{E}}]$. The transform of the corresponding compensation measure at the boundary $\tilde{\boldsymbol{D}}$ is then given by

$$K\left[-1+\frac{\tilde{\lambda}}{\tilde{\lambda}-w}\right] e^{-w\tilde{\boldsymbol{D}}} = K\left[\frac{w}{\tilde{\lambda}-w}\right] e^{-w\tilde{\boldsymbol{D}}}$$

where K is a positive constant to be determined. To understand the form of the compensation, one observes it to be equivalent to a delta function at x=O with transform -1 and an exponentially distributed mass with transform $\frac{\tilde{\lambda}}{\tilde{\lambda}-w}$ and a shift of $+\tilde{\boldsymbol{D}}$ corresponding to $e^{-w\tilde{\boldsymbol{D}}}$.

There is a similar compensation mass localized at the boundary at 0. There one must have negative mass to annihilate the overshoot mass on (-∞,O) and positive mass concentrated at 0 representing the entry into the upper part of the state space $\tilde{S}$ after passing through the idle state. The renewal rate for departures from stockout is $\tilde{\lambda} e_\infty$. The transform of the compensation measure at 0 is then of the form , $\tilde{\lambda} e_\infty \left[1-\frac{\tilde{\lambda}}{\tilde{\lambda}-w}\right] = \tilde{\lambda} e_\infty \left[\frac{w}{\tilde{\lambda}-w}\right]$ and the transform of the total compensation measure is

$$\chi_\infty(w) = -\tilde{\lambda} e_\infty \left[\frac{w}{\tilde{\lambda}-w}\right] + K\left[\frac{w}{\tilde{\lambda}-w}\right] e^{-w\tilde{\boldsymbol{D}}}$$

with K positive and yet to be determined. The negative exponentially distributed compensation from the $\tilde{\boldsymbol{D}}$ boundary which overshoots 0 has the same structure as the compensation at 0 and blends with it since the arrival rate exceeds the perishing rate.

2.5 The probability of having an empty stock

The compensation method states that the ergodic distribution on S is the convolution of the compensation and the ergodic Green's function. With the convention that X(t) = 0 when the system is empty, the ergodic distribution of X(t) which will be designated by X_∞, has the transform.

$$\varphi_\infty(w) = e_\infty + \chi_\infty(w)\, \gamma_H(W,\infty)$$

$$= e_\infty + \frac{-\tilde{\lambda} e_\infty \left[\frac{\boldsymbol{w}}{\tilde{\lambda}-\boldsymbol{w}}\right] + K\left[\frac{\boldsymbol{w}}{\tilde{\lambda}-\boldsymbol{w}}\right] e^{-w\tilde{\boldsymbol{D}}}}{\tilde{\boldsymbol{\mu}}\left(1-\frac{\tilde{\lambda}}{\tilde{\boldsymbol{\mu}}}\right)+w}$$

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where e_∞, = P[X(∞) = 0]. Note that X_∞, has all support on $[0, \tilde{D}]$ so that $\varphi_\infty(w)$ is entire. The zero in the denominator must be counteracted by a zero in the numerator. One also has $\varphi_\infty(0)$= 1. Algebra then gives

$$\varphi_\infty(w) = e_\infty + \tilde{\lambda}\, e_\infty \frac{1-e^{-w(\tilde{\lambda}-\tilde{\mu})\tilde{D}}\, e^{-w\tilde{D}}}{w-\tilde{\lambda}+\tilde{\mu}} \quad \text{with}$$

$$e_\infty = \frac{\tilde{\lambda}-\tilde{\mu}}{\tilde{\lambda}e^{-w(\tilde{\lambda}-\tilde{\mu})\tilde{D}} - \tilde{\mu}}$$

It should be noted that $0 < e_\infty < 1$ whether or not $\tilde{\lambda} > \tilde{\mu}$; When $\tilde{\lambda} = \tilde{\mu}$. then

$$e_\infty = \frac{1}{1+\tilde{\lambda}\tilde{D}}$$

Property 2: When $\tilde{\mu}$=(0,0,0), one has $e_\infty = e^{-\tilde{\lambda}\tilde{D}}$

This result is true for any selection policy, when there is no demand (i.e. $\tilde{\mu}$=(0,0,0)), as one can see from M/G/∞ when each item arriving is given its own shelf (server) and awaits outdating.

The probability that the system is not empty is clearly

Service Level $1 - e_\infty = 1 - \frac{\tilde{\lambda}-\tilde{\mu}}{\tilde{\lambda}e^{-w(\tilde{\lambda}-\tilde{\mu})\tilde{D}} - \tilde{\mu}}$

$$= \frac{\tilde{\lambda}e^{-w(\tilde{\lambda}-\tilde{\mu})\tilde{D}} - \tilde{\lambda}}{\tilde{\lambda}e^{-w(\tilde{\lambda}-\tilde{\mu})\tilde{D}} - \tilde{\mu}}$$

The spoilage rate is $\tilde{\lambda} - \tilde{\mu}$**P[not empty]** , simple algebra gives

Spoilage Rate = $\tilde{\lambda}\frac{\tilde{\lambda}-\tilde{\mu}}{\tilde{\lambda}-\tilde{\mu}e^{(\tilde{\lambda}-\tilde{\mu})\tilde{D}}}$

Note that the ratio of spoilage rate to arrival rate is a positive fuzzy number smaller than one as it should be. This ratio decreases with $\tilde{D}$ and decreases with $\tilde{\mu}$.

2.6 The distribution of the age of the items delivered:

The age of the item delivered in the steady state is the conditional distribution of $\tilde{X}_\infty$, for $\tilde{X}_\infty$, , positive. One then has from φ_∞(w), with $\tilde{\theta} = \tilde{\lambda} - \tilde{\mu}$,

$$\varphi(\tilde{X}_\infty, |\tilde{X}_\infty, > 0)(w) = \frac{\frac{1-e^{(\tilde{\theta}-w)\tilde{D}}}{w-\tilde{\theta}}}{\frac{1-e^{\tilde{\theta}\tilde{D}}}{-\tilde{\theta}}}$$

The expected value of the age of items delivered is then

$$E(\tilde{X}_\infty, |\tilde{X}_\infty, > 0) = -\varphi'(\tilde{X}_\infty, |\tilde{X}_\infty, > 0)(0)$$

$$= \tilde{D}\frac{d}{dv}\log\frac{e^v-1}{e^v} \text{ / at } v = \tilde{\theta}\tilde{D}$$

$$= \tilde{D}[1-v+(e^V-1)^{-1}].$$

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Conclusion:

Thus in this paper some of the performance characteristics of perishable inventory systems are derived for FIFO policy are evaluated theoretically.